Model Risk Management: Quantifying, Monitoring and Mitigating Model Risk

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Model Risk Management

Massimo Morini
Agenda

- **What is Model Risk**
- Model Risk in Past Crises
- Practical Examples
  - In Credit Modelling
  - In Interest Rate Modelling
  - In Equity Modelling
- Managing Model Risk:
  - Model Reserves
  - Model Limits
  - Model Revisions
- Model Risk from 3 points of view
  - Mathematics
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- Art & Science of Model Validation
What is Model Risk?

The Value Approach

“Model risk is the risk that the model is not a realistic (or at least plausible) description of the factors that affect the derivative’s value”

By E. Derman

VS

The Price Approach

“Model risk is the risk of a significant difference between the mark-to-model value of an instrument, and the price at which the same instrument is revealed to have traded in the market”

By R. Rebonato
What is model risk?

- There are two main approaches to Model Risk. One dates back the words of Derman (1996, 2001). A model risk manager should check the following points:

1. Is the payoff accurately described?
2. Is the software reliable?
3. Has the model been appropriately calibrated to the prices of the simpler, liquid constituents that comprise the derivative?
4. Does the model provide a realistic (or at least plausible) description of the factors that affect the derivative's value?
The Value Approach

“Model risk is the risk that the model is not a realistic (or at least plausible) description of the factors that affect the derivative's value”

By E. Derman

- It is clear that “a model is always an attempted simplification" of reality, and as such there can be no perfectly realistic model. Moreover only what affects the derivatives’s value matters. Yet realism remains a goal, and modellers should avoid the following errors:

  “You may have not taken into account all the factors that affect valuation. You may have incorrectly assumed certain stochastic variables can be approximated as deterministic. You may have assumed incorrect dynamics. You may have made incorrect assumptions about relationships".

  By E. Derman

- But Rebonato starts from a toally different perspective…
What is Model Risk?

The Price Approach

“Model risk is the risk of a significant difference between the mark-to-model value of an instrument, and the price at which the same instrument is revealed to have traded in the market”

By R. Rebonato

- Real losses do not appear “because of a discrepancy between the model value and the ‘true’ value of an instrument”, but through the mark-to-market process, because of a discrepancy between the model value and the market price of an instrument.
- As long as the market agrees with our valuation, we do not have large losses due to models, even if “market prices might be unreasonable, counterintuitive, perhaps even arbitrageable”.
- We have no mark-to-market losses, we can sell at the value at which we booked.
Are Value and Price approaches really so different?

The Price approach rightly stresses that model losses usually emerge when a sudden gap opens between market price and model booking.

But this can happen for 3 reasons

1. **We are using a different model from market consensus**

   **Market Intelligence: trades, collateral, TOTEM, rumors…**

2. **Same model, but implementation or payoff are wrong**

   **Price Verification: check software, legal issues…**

3. **Same model, but the market consensus suddenly changes**

   “Surmise how today’s accepted model can change in the future…”
Let’s look at the subprime crisis

- **How can the market suddenly change the model consensus?**
  This happened recently for mortgage-backed securities.

- The market consensus modelled the default probabilities in a pool of subprime mortgages based on a Gaussian Copula with correlations that were historically estimated. Such correlations were low, allowing for low risk estimates and high ratings for senior CDO tranches.

- Historical correlations and default probabilities were low since in the previous decade subprime defaults had led to few and sparse losses. This was due to the increasing trend of house prices, giving cash to mortgagers and a valuable guarantee to the bank in case mortgager defaulted.

- Using historical correlations for the future meant assuming implicitly that house prices were going to grow at same rate also for the subsequent decade.
How can market consensus on models change suddenly?

Example 1: Mortgage CDOs in the Subprime Crisis

Model consensus was Gaussian Copula with estimated/mapped correlations, very low consistently with the assumption of ever-increasing national house prices.

The reversal of the national house trend reveals that mortgage losses can be very correlated, and the Gaussian Copula market consensus collapses.
How can market consensus on models change suddenly?

Example 2: 1987 Stock Market Crash

Example 3: From One factor to Multifactor in 2000

“How to throw away a billion Dollar”

\[
\begin{align*}
\text{Example 3: From One factor to Multifactor in 2000} & \\
\text{“How to throw away a billion Dollar”} & \\

\text{The formulae for the multifactor model are:} & \\

& dF_1(t) = \mu_1(t) \, dt + \sigma_1(t) \, F_1(t) \, dW_1^t \\
& dF_2(t) = \mu_2(t) \, dt + \sigma_2(t) \, F_2(t) \, dW_2^t \\
& \vdots \\
& dF_n(t) = \mu_n(t) \, dt + \sigma_n(t) \, F_n(t) \, dW_n^t
\end{align*}
\]
A synthetic view on Model Risk

- In the subprime crisis and in 1987 crash it is an event in the fundamentals to change the model consensus. Something happens in the reality of the markets that reveals an element of unrealism of the model to be more relevant than previously thought. In the third example it is a piece of research that shows some of the modelling assumptions to be unrealistic in a relevant way.

- Thus realism and reasonableness of the model are relevant not only in Derman’s Value approach, but also in Rebonato’s Price approach to model risk management, since lack of consistency with reality is the main fact that can lead to sudden changes in model consensus.

- On the other hand, consistency with market consensus, the main point in the Price approach, is not necessarily overlooked in the Value approach. Among the factors to be considered in designing a plausible model, Derman’s Value approach includes market sentiment.
A first scheme for model choice, validation and risk management

- **Model Verification**
  - Mathematics
  - Implementation
  - Numerics
  - Correct application to Payoff

- **Model Validation**
  - Calibration
  - Reasonableness
  - Market Intelligence
  - Reality check

The last two points make sense if applied in **Model Comparison**. This leads to model selection but also the setting of **provisions** (reserves, model limits, monitors..) for residual **model uncertainty**.
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Model Comparison for Model Validation

Classic examples of Model Comparison in the literature are:

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<tr>
<td>Structural Models</td>
<td>VS</td>
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</tbody>
</table>
Comparing models: an example from Credit Modelling
**Structural First-Passage models vs Reduced-form Intensity Models**

**Structural Models**

\[ dV_t = (r - q) V_t dt + \sigma V_t dW \]

\[ \tau = \inf \{ t \mid V_t \leq H_t \} \]

V is the firm value, H is the default barrier.

**Intensity Models**

\[ \tau = \inf \left\{ t : \int_0^t \lambda(s) \, ds \geq \varepsilon \right\} \]

\[ \Pr(\varepsilon \leq z) = 1 - e^{-z}. \]

\( \lambda(t) \) is the intensity or instantaneous default probability, giving credit spreads, \( \varepsilon \) is the impredicable default trigger.
THE PRODUCT: A leveraged note with a trigger

Consider a simple note sold by a bank to a client. The client pays 1, with this notional the bank sells leveraged protection on a reference entity. The notional of this sale is $Lev$. If a default happens, the bank may have a loss given by

$$Lev \times Lgd - 1$$

A typical way to mitigate this gap risk is to set a trigger to stop the note if spreads are growing too much:

The trigger can be set at such a level $trigger^*$ that when touched, if unwinding is timely, we expect to have a loss around 1, minimizing gap risk.
1. Calibration

Both models must be able to calibrate the CDS term structure of the reference entity. This is natural for standard (deterministic) intensity models, while structural models must be upgraded to time-dependent parameters and barrier (i.e. Brigo and Morini (2006))

2. Reasonableness

Both models must admit stochastic credit spreads, since spread dynamics is crucial for gap risk. This is natural for structural models, while intensity must be upgraded to stochastic intensity (i.e. Lando (1999)).

\[ d\lambda(t) = k[\theta - \lambda(t)]dt + \sigma\sqrt{\lambda(t)}dW(t) \]
Are now the two families of models equivalent for Gap Risk?

- Big gap losses are associated to entity defaulting without touching the trigger first. Thus gap risk depends crucially on the behaviour of spreads in the time preceding default: will default be preceded by a significant rally in the spread, or we will have an abrupt leap to default?

A) Default anticipated by spread rally: if the rally is not too fast, the deal is stopped with no gap losses.

\[ \text{GapRisk} \approx 0. \]

B) Sudden leap to default, not sufficiently anticipated by spread rally: the bank has the maximum possible gap risk, given by

\[ \text{GapRisk} = \text{Discount} \times \Pr(\tau < T_M) \times (\text{Lev} \times (1 - \text{Rec}) - 1)^+ \]
Comparing Structural Models and Intensity Models

**Structural Models**

Credit spreads grow with default probability. In standard structural models default probabilities are higher the closer Firm Value gets to the Barrier, so credit spreads rally to infinity when default approaches. In a trigger note this means **LOW GAP RISK**.

**Intensity Models**

In intensity models, if the intensity is a stochastic diffusion - CIR for example – in most cases default is a sudden leap not anticipated by spreads. In a trigger note it is **HIGH GAP RISK**.
Why Gap Risk is so high (leap to default) in intensity models?

The blue line is simulated diffusive (CIR) intensity.

The green line is its own integral.

Default is when integral reaches simulated $\varepsilon$. With these patterns, it is $\varepsilon$ that decides default, and $\varepsilon$ is independent of spreads.
Adding Market Information: CDS Options?

- Do we have liquid quotes to make a choice? CDS do not help, because the above models are both perfectly calibrated to CDS.

- One possibility could be CDS options, when available. But a CDS option is knock-out, so if maturity is $T$ and strike is $K$ we have in the payoff

$$1_{\{\tau>T\}} (S(T) - K)^+$$

thus it depends crucially on

$$\Pr (S(T) > K | \tau \geq T)$$

while for a note with trigger = $K$ and same maturity $T$ we are interested in

$$\Pr (S(\tau) > K | \tau < T)$$

- The two derivatives speak of two opposite states of the world! One is conditional to survival, the other one on default. Options increase our information but are not crucial for model choice when we have to evaluate a trigger note.
Realism: in search of historical evidence

- Do we have a clear historical evidence of what is the most likely possibility, a leap to default or a default preceded by a spread increase?
- The first caveat is that history of past defaults has little to do with future defaults.
- Moreover the history of past defaults is mixed.

In general, the likelihood of a sudden default is inversely proportional to the transparency of the balance sheet of the reference entity, but...

Parmalat and Enron were little predictable. Argentina, a sovereign default, was fully predictable, with spreads growing steadily. Lehman was mixed.
Market Intelligence and the practical side of Incompleteness

- The leveraged note depends crucially on a risk factor, driving the relation between spreads and default time, which is not observable in any liquid market. This has two related consequences:

- We have no quotes to be used to calibrate the model to market consensus about the behaviour of this risk factor. One may think of market intelligence...

- The market on this risk factor is incomplete. In an incomplete market, even if all players agreed on the actual probability to have a sudden leap-to-default rather than a spread rally anticipating default, they may disagree on the compensation for the risk of this event. A conservative bank selling this note, for example, will increase the risk-adjusted probability of such a leap to default.

Lack of Historical Evidence

High Model Risk

There is no visible market consensus

Different risk aversions influence price
Dealing with Model Uncertainty

- How to deal with a situation where model uncertainty is so high, and the two most popular modelling solution give two opposite values to the derivative?

- First, find models with behaviour in-between the “null” Gap Risk of standard Structural Models and the “maximum” Gap Risk of the Intensity Models seen. There are at least 2 possibilities.

1. More realistic structural models, with a default barrier that can jump between different levels. An unpredictable barrier creates in these models a controllable probability of leaps to default.

2. Intensity models where intensity stops being a diffusion:

\[ d\lambda(t) = k[\theta - \lambda(t)]dt + \sigma\sqrt{\lambda(t)}dW(t) + dJ_t^{\alpha,\gamma} \]

where \( dJ_t^{\alpha,\gamma} \) is a Poisson jump with intensity \( \alpha \) and jumps which are exponentially distributed with average size \( \gamma \).
How can intensity jumps create predictable defaults?

The blue line is simulated intensity CIR with jumps.

The green line is its own integral.

Default is when integral reaches $\varepsilon$.

Here it is not $\varepsilon$ that decides default, but the jump in the intensity, which first creates a spread rally anticipating default.
Default times and spreads when intensity is jump-diffusion
A parametric family of models covering the possible values

In this family of models, moving the expected jumps size we move expected loss from maximum to almost zero.

How to use parametric family for model risk management?
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Model Risk Management: Reserves, Limits, Revisions
Suppose for simplicity a class of models parameterized by the parameter $\gamma$, $0 \leq \gamma \leq 1$.

The value

$$\Pi_t^T (\gamma)$$

of the derivative increases with $\gamma$.

The model validation process validates the model with $\gamma=0.5$ but recognizes that there is a residual model uncertainty.

If counterparty is eager to buy the derivative at $\gamma=0.75$, the difference

$$R = \Pi_t^T (0.75) - \Pi_t^T (0.5)$$

appears a day-0 profit for the salespeople and the traders that closed the deal.

The model risk managers can instead use $R$ to create a model capital reserve, that will be released to the trader only along the life of the derivative, for example from time $t_1$ to $t_2$ he may linearly receive only

$$R \frac{(t_2 - t_1)}{T}$$
Model risk management: Model Reserves

- A reserve can be set up also for products which are bought. In general, with γ^fair being the parameter used for fair value and γ^deal being the one used for the deal, the general formula is

\[
(\Pi_t(γ^fair) - \Pi_t(γ^deal))^+
\]

- The reserve is updated and potentially released at maturities and for new deals, at the level of the portfolio priced with the “risky” model.

- They can also be updated for changes in market values and certainly for changes to γ^fair, by tracking the change in Π(γ^fair).

- The release policy must take this into account.
Model risk management: Model Lines

- Model *position limits* or *lines* are analogous to the credit lines used to manage credit risk. If a model is considered subject to high model risk, the bank can set a limit to the exposure built through the model.

1. First decide first the **total model line** - the maximum exposure to model uncertainty allowed.

2. Then compute the how much a single deal contributes to filling the total line. It is the potential loss due to model uncertainty.

- For a product sold at \( \Pi_t (\gamma^\text{deal}) \) an estimate is

\[
\text{Notional} \times \left[ \Pi_t^T (1) - \Pi_t^T (\gamma^\text{deal}) \right]
\]

- The line does not regard deals sold at the conservative price \( \Pi_t^T (1) \)
Notice that the above formula is valid if \( \gamma^{\text{deal}} = \gamma^{\text{fair}} \) or if there is a model reserve. In general, and for the total portfolio priced with the “risky” model, the formula for the amount of line already eaten out is

\[
\text{Notional} \times \left[ \Pi_t^T (\gamma^{\text{fair}}) - \min_{\gamma \in [0,1]} \Pi_t^T (\gamma) \right].
\]

since this is the amount at stake due to model risk.

Then, if there is a model reserve this can be subtracted from the above amount.

If the error we expect on \( \gamma \) is no more than \( \Delta \gamma \), one can use

\[
\text{Notional} \times \left. \frac{\partial \Pi_t^T (\gamma)}{\partial \gamma} \right|_{\gamma = \gamma^{\text{fair}}} \Delta \gamma.
\]
Model risk management: Model Revisions

- For credit lines the exposure is often a quantile of the future mark-to-market. Practically this is undesirable since, with uncertainty on \( y \), the quantile is less reliable. Theoretically, model risk is not associated to uncertainty about future values – market risk - but about present prices.

- **Model Revisions**: specify dates at which the validation must be revised.

  1. Periodic Revisions: scheduled regularly, for example yearly. After one year experience and evidence have increased.

  2. Triggered Revisions: stress-test may have revealed that under particular market conditions a model may become unreliable. If quantitative triggers signal such conditions are reached, validation must be revised.

What’s the theory behind this model risk management practice?
Model Comparison for Model Validation

Classic examples of Model Comparison in the literature are:

- **Equity**
  - Stochastic Volatility vs Local Volatility

- **Rates**
  - Low factor Short Rate vs Multifactor BGM/HJM

- **Credit**
  - Structural Models vs Reduced-form Models
Comparing models: an example for Equity
Local vs Stochastic Volatility Models

Stochastic Volatility

\[ dS = rSdt + \sigma \sqrt{V} SdW \]
\[ dV = k(V - \theta) dt + \nu \sqrt{V} dZ \]

Local Volatility

\[ dS(t) = r(t) S(t) dt + \Sigma(S(t), t) S(t) dW^s(t) \]

We can force them to have the same Marginal distributions

\[ Pr(S(T) \in [x, x + dx) | S(0)) \]

But transition distributions still differ strongly

\[ Pr(S(T_2) \in [x_2, x_2 + dx) | S(T_1) = x_1) \]
The simplest example on marginal and transition densities

Sort of Stoch Vol?
\[
X^A(0) = X(0) \\
X^A(t_1) = \mathcal{N}(0, 1) \\
X^A(t_2) = \mathcal{N}(0, 1) \perp X^A(t_1) \\
X^A(t_3) = \mathcal{N}(0, 1) \perp X^A(t_2) \perp X^A(t_1) \\
\vdots
\]

Sort of Local Vol?
\[
X^B(0) = X(0) \\
X^B(t_1) = \mathcal{N}(0, 1) \\
X^B(t_2) = X^B(t_1) \\
X^B(t_3) = X^B(t_2) = X^B(t_1) \\
\vdots
\]

Same Marginals

Totally different transition densities
How relevant it is? What the market believes, and what in reality?

- Less than 2% difference on compound option (1 bivariate transition density)
- More than 50% difference on barrier options (many multivariate transition densities)

- Local Volatility used to be market standard, shaken by some results (Hull and Suo on asset-volatility correlation, Hagan et al. on hedging)
- Stochastic Volatility is not opposite assumption. Models admitting jumps have a lot to say on this model risk.
Comparing models: an example for Rates
Low factor Short Rate vs Multifactor BGM/HJM

- Popular debate around 2000. The paper “How to throw away a billion dollar” claims bermudan swaptions are undervalued by 1-factor interest rate models, that have have instantaneous correlations among rates set to 1.

\[ dr_t = k (\theta - r_t) \, dt + \sigma dW_t \]

- This can be an element of model risk. Think of a montecarlo simulation. If correlation among the different payoffs you can get exercising is forced to be high, even as high as one, then having Bermudan rather than European is not big deal. Bermudans will have low values.

- Compared to real world correlations, mipic strategies.
Low factor Short Rate vs Multifactor BGM/HJM

- When correlations are allowed to be lower, in any scenario there can be low exercise values at some times and higher exercise values at other times. With a Bermudan you can find convenient exercise time that you could not find with one European.

- It seems that for getting this one needs multifactor models with lower correlations.

\[
dS_{a,b}(t) = \mu_a(t) \, dt + \sigma_a(t) \, dW_a(t),
\]
\[
dS_{a+1,b}(t) = \mu_{a+1}(t) \, dt + \sigma_{a+1}(t) \, dW_{a+1}(t),
\]
\[
\vdots
\]
\[
dS_{b-1,b}(t) = \mu_{b-1}(t) \, dt + \sigma_{b-1}(t) \, dW_{b-1}(t).
\]
However, when pricing Bermudans…

It is **SERIAL** correlations that matter, not **INSTANTANEOUS** correlations.
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Model Uncertainty, Incompleteness and Illiquidity
What is Model Uncertainty?

- The analysis of model realism and the intelligence about the models used in the market does not always eliminate model uncertainty. How should we deal with this uncertainty? What is model uncertainty?

- Brigo and Mercurio 2003 formalized a so-called Uncertain Volatility Model where

\[
dS(t) = rS(t) \, dt + \sigma^I S(t) \, dW(t)
\]

\[
\sigma^I = \begin{cases} 
\sigma^1 \text{ with prob } p_1 \\
\sigma^2 \text{ with prob } p_2 
\end{cases}
\]

- Scenarios depend on random variable \( I \), drawn at \( t=\varepsilon \) infinitesimal after 0, independent of \( W \), taking values in \( 1,2 \) with probability

\[
Prob(I = i) = p_i.
\]
What is Model Uncertainty?

- How does one price the option with strike \( K \) and maturity \( T \)? One uses the law of iterated expectation

\[
\Pi (K, 0, T) = P(0, T) \mathbb{E} \left[ (S(T) - K)^+ \right]
\]

\[
= P(0, T) \mathbb{E} \left[ \mathbb{E} \left[ (S(T) - K)^+ | \sigma^I \right] \right]
\]

\[
= P(0, T) \sum_{i=1}^{2} p_i \mathbb{E} \left[ (S(T) - K)^+ | \sigma^I(t) = \sigma^i(t) \right].
\]

- The option price is just the average of two Black and Scholes prices

\[
p_1 BS \left( S(0), K, T, \sigma^1 \right) + p_2 BS \left( S(0), K, T, \sigma^2 \right)
\]

- Is this an example of model uncertainty? NO. Here there is only ONE model, with a sketchy random volatility. Volatility is a random variable with a simple distribution which must be drawn before simulating \( S \).
What is Model Uncertainty?

- Model Uncertainty looks like
  \[ dS(t) = rS(t)\, dt + \sigma S(t)\, dW(t) \]
  with us not knowing if \( \sigma = \sigma^1 \) OR \( \sigma = \sigma^2 \)

- What has changed from above?

  - at \( \varepsilon \) we will not draw the right volatility
  - we have no idea of what \( p_1 \) and \( p_2 \) are
  - other market players may know what is the real value of \( \sigma \) or they may have even less information than us.
Model Uncertainty vs Risk

- Cont (2006) treats model uncertainty as multiple probability measures

\[ (\Omega, \mathcal{F}, P_i \mapsto Q_i) , \]

\[ P_i \mapsto Q_i = P_1 \mapsto Q_1, \ldots, P_N \mapsto Q_N \]

as opposed to risk, where we are uncertain about realizations but we know the probability distributions (the roulette for a standard player).

- The Bayesian approach averages out expectations under different measures, treating uncertainty \textit{about} the model in the same way as uncertainty \textit{within} the model, in spite of the above differences.
The conservative approach glorified

- Cont (2006) notices that the typical approach of banks is not to average across models but to adopt a worst case approach. Only one choice protects you from any model loss: with $\sigma^2 > \sigma^1$,

$$P^{ask} = BS\left( S(0), K, T, \sigma^2 \right), \quad P^{bid} = BS\left( S(0), K, T, \sigma^1 \right).$$

- Gibdoa and Schmeider (1989) show that this old `conservative approach' is rational maximization of utility when we are faced with total ignorance on the probabilities.

- Following this line, Cont (2003) proposes two measures of model uncertainty: one is akin to

$$\max_{Q_i=1,\ldots,N} \text{Price} - \min_{Q_i=1,\ldots,N} \text{Price},$$
Model Risk and Liquid Markets

- The second one weights more or less models depending on the higher or lower capability to price liquid market instruments.

- For Cont there is no model uncertainty when:
  1. the **market is liquid** (model uncertainty lower than bid-ask)
  2. we can set up a **model-independent static hedge**

- In principle, this makes sense. But in practice? Should we forget Model Risk management for liquid/hedgeable products?

- We are going to see first that accountancy boards seem to agree, then we will see an example where the collapse of a model consensus shook up a liquid market, disrupted standard practice for discounting, broke up hedging and turned the market illiquid. There will see that and apparent “model independent” static hedge can be broken down by a sudden change of model consensus.
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Accountants, Regulators and the explosion of credit and liquidity risk
Accountancy for Modellers

- There are various aspects of accountancy principles that are linked to the debate about model risk and model uncertainty.

  **Fair Value**
  
  "Fair value is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction" (FAS 157)

- This reminds of the Price approach…market is the real reference.

- But in the crisis the "SEC Office of the chief accountants and FASB Staff Clarification on Fair Value Accounting“ says that at times illiquidity, panic or euphoria do not allow for "orderly transactions", so that market prices can be corrected by personal judgment to get

  "reasonable risk-adjusted, liquidity-adjusted and credit-adjusted expected cash flows”

- This reminds much more of the Value approach…
Level 1, 2, 3… go?

- Another analogy are the famous “levels” for prices:

1) **Level 1 (Liquid specific quotes):** the price comes from a liquid market where products identical to the derivative to price are traded.

2) **Level 2 (Comparable quotes or Illiquid quotes):** Prices come from either:
   a) a liquid market where products similar (not identical) to the derivative to evaluate are traded.
   b) a market where products identical to the derivative to evaluate are traded but the market is illiquid.

3) **Level 3 (Model with non-quoted parameters):** Prices come from the a valuation technique that requires inputs which involve a crucial amount of personal judgement of the institution computing the price. This kind of prices are often called 'marked-to-model'.
Level 2 and Level 3

- Standard market wisdom believes that:
  \[ \text{ModelRisk(Level3)} > \text{ModelRisk(Level2)} > \text{ModelRisk(Level1)} = 0 \]

- Are we sure?
  
- Example 1:
  - Realistic model: two factors with a correlation to be estimated: Level 3
  - Unrealistic models: one factor (correlation=1) or two factors independent by construction (correlation=0): Level 2

- A trader, if penalized by model reserves in case of Level 3 prices, has incentive to choose the most simplified and unrealistic model.

- And what about Level 1 prices, are they really model-risk-free?
Level 1 and model risk

- Level 1 prices are those taken from liquid prices of products identical to the derivative under analysis.

- A Level 1 price can involve model risk because it hinges on the subjective decisions that two assets are identical, while this may be true only under some hidden assumption that can suddenly become unrealistic.

- For pointing out how model assumptions enter in the evaluation of the simplest vanilla derivatives we show the model assumptions hidden in the valuation of a simple swap. By the way, during the credit crunch – on 9 August 2007 for precision - these assumptions have broken down, opening a radical reform of pricing.
Swaps are the main interest rate derivatives. They are priced ‘without a model’ via replication.

The simplest swap is the Forward Rate Agreement (FRA). We can price it using information in the spot interbank market, where loans are made from bank to bank.

If the prevailing rate is $L(t, T)$, when a lender gives 1 at $t$ he receives

$$1 + L(t, T)(T - t) \text{ at } T$$

If instead he gives

$$\frac{1}{1 + L(t, T)(T - t)} \text{ at } t$$

he receives 1 at $T$. 

When the market model changes. Example from Rates
Replicating a FRA

\[
\begin{align*}
0 & \quad 0 \\
- \frac{1}{1 + \frac{1}{2} (0, \alpha) \alpha} & \quad + 1 \\
\frac{1 + K \alpha}{1 + \frac{1}{2} (0, 2\alpha) \alpha} & \quad - 1 - K \alpha \\
\alpha & \quad 1 + L (\alpha, 2\alpha) \alpha \\
2\alpha & \quad (L (\alpha, 2\alpha) - K) \alpha
\end{align*}
\]
Replicating a FRA

- If a FRA fixes in $\alpha$ and pays in $2\alpha$, the payoff in $2\alpha$ is:
  \[
  (L(\alpha, 2\alpha) - K) \alpha.
  \]

- To replicate it, lend at 0 to your counterparty until $\alpha$, so that you get 1 at $\alpha$. At $\alpha$ lend again to counterparty 1 until $2\alpha$, getting at $2\alpha$:
  \[
  1 + L(\alpha, 2\alpha) \alpha
  \]

- At 0 you also borrow \[
  \frac{1 + K\alpha}{1 + L(0, 2\alpha) \alpha}
  \]
  until $2\alpha$, so that you receive at $2\alpha$:
  \[
  -1 - K\alpha
  \]

- Putting together, payoff at $2\alpha$ is:
  \[
  (L(\alpha, 2\alpha) - K) \alpha.
  \]
The equilibrium FRA rate and the Basis Spreads

- The price of a FRA is the cost of its replication. Recalling that in modern markets the discount “bond” comes actually from interbank rates:

\[
\frac{1}{1 + L(0, \alpha) \alpha} - \frac{1 + K\alpha}{1 + L(0, 2\alpha) \alpha} = P(0, \alpha) - P(0, 2\alpha) - P(0, 2\alpha) K\alpha
\]

and the equilibrium K is:

\[
F(0; \alpha, 2\alpha) = \frac{1}{\alpha} \left[ \frac{P(0, \alpha)}{P(0, 2\alpha)} - 1 \right]
\]
The equilibrium FRA rate and the Basis Spreads

- With many FRA paying from $T_{a+1}$ to $T_b$ we have swap value:

$$
\sum_{i=a+1}^{b} \left[ P(t, T_{i-1}) - P(t, T_i) - P(t, T_i) K \alpha_i \right] = P(t, T_a) - P(t, T_b) - \sum_{i=a+1}^{b} P(t, T_i) \alpha_i K
$$

- The frequency of payment, and consequently the tenor of the rates paid, does not enter the valuation of floating legs.

- This implies null spreads for Basis swaps, that exchange two floating legs of different frequency and different rate indexation (for example 1y vs 6m).
9 August 2007: the market changes

- Is this pricing Level 1, 2 or 3?

- There are no estimations based on judgements, we are not using comparables but a strategy that yields exactly the same payoff. It should be level 1.

- In spite of this…
9 August 2007: the market changes

None of these relationships holds anymore. Below see spot replication of 6mX6m FRAs vs real market FRAs.
9 August 2007: the market changes

- Here we see the dynamics of Basis Spreads in the crisis, to be compared with null spread predicted by models.

- Old relations were valid for decades but were rejected in one day.
Introducing Reality

- Which model hypothesis is not valid anymore? The usual suspect is the implicit hypothesis that

   The interbank market is free of default risk

- Default risk can break the above replication. If each bank has its own risk of default, it also has its own fair funding rate, and there is no reason to expect the fair funding rate of my counterparty at $\alpha$, $L_{\text{Counterparty}}^{\alpha, 2\alpha}$, to be the same as the payoff rate $L(\alpha, 2\alpha)$.

- We have used this hypothesis in replication.

- The payoff rate is usually Libor... a very complex basket payoff!

Libor is a trimmed average of reported funding rates for unsecured borrowing deals from a panel of large banks with the highest credit quality.
Replicating a FRA after the Credit Crunch

\[ 0 \quad 0 \quad 0 \]

\[ - \frac{1}{1 + \zeta(0, \alpha) \alpha} \quad \frac{1}{1 + \zeta(0, 2\alpha) \alpha} \quad 1 \]

\[ +1 \quad -1 \quad 1 + \zeta(\alpha, 2\alpha) \alpha \]

\[ \mathcal{L}^C(\alpha, 2\alpha) \]

\[ -1 - K \alpha \]

\[ (\mathcal{L}(\alpha, 2\alpha) - K) \alpha \]
An option hidden in Libor…

- However, understanding how to explain the new market with a new model is not easy
- In the Libor (or Euribor) markets there is a selection mechanism that creates a bias towards rates with lower risk.

- In fact, banks whose funding rate grows too much, sooner or later…
  - will exit the Libor interquantile average
  - will exit the Libor market and start dummy contributions
  - will exit the Libor panel
  - will exit this world (and Libor) by defaulting…

- Thus the expected future Libor rate is constantly lower than the expected future borrowing rate of today’s Libor counterparty
A simple scheme to represent the potential refreshment of counterparty is defining the Libor credit spread as

\[
S(\alpha, 2\alpha) = \begin{cases} 
S_{Initial}(\alpha, 2\alpha) & \text{if } S_{Initial}(\alpha, 2\alpha) \leq S_{Exit} \\
S_{Subst} & \text{if } S_{Initial}(\alpha, 2\alpha) > S_{Exit}
\end{cases}
\]

- \(S_{Exit}\) is the maximum level of the spread for the initial counterparty to be still a Libor counterparty at \(\alpha\)
- \(S_{Subst}\) is the spread of Libor in case the initial counterparty is no more a Libor bank because

\[
S_{Initial}(\alpha, 2\alpha) > S_{Exit}
\]
The Libor mechanics: possible substitution and exit condition

\[
\mathbb{E} [L (\alpha, 2\alpha)] = F_{Libor} (0; \alpha, 2\alpha) - 2\text{BlackCall} (S_{Initial} (0; \alpha, 2\alpha), S_{Initial} (0; \alpha, 2\alpha), \sigma_\alpha \sqrt{\alpha})
\]
An option in Libor…

- We need a volatility. We take it from i-Traxx credit options. We can explain market patterns better than with old models.
Agenda

- What is Model Risk
- Model Risk in Past Crises
- Practical Examples
  - In Credit Modelling
  - In Interest Rate Modelling
  - In Equity Modelling
- Managing Model Risk:
  - Model Reserves
  - Model Limits
  - Model Revisions
- Model Risk from 3 points of view
  - Mathematics
  - Accountancy
  - Regulations
- Art & Science of Model Validation
What Regulators Say
Basel Committee Supervisory Guidance on Fair Value:

- Validation includes evaluations of the model's theoretical soundness and mathematical integrity and the **appropriateness of model assumptions**, including **consistency with market**.

- Bank processes should emphasise the importance of assessing fair value using a **diversity of approaches** and having in place a range of mechanisms to cross-check valuations.

- A bank is expected to test and review the performance of its valuation models under possible **stress conditions**, so that it **understands the limitations of the models**.

- Assess the **impact of variations in model parameters** on fair value, including under **stress conditions**.
Stress Testing

- **Stress-testing**
  - Stressability: Verify if the model can be used to express stress-conditions
  - Stress-testing implementation: detect what can impair the precision of approximations and numerics
    - for market conditions
    - for payoff features
  - Stress testing assumptions: subject the model to stress conditions to limit and monitor its application
    - for market conditions
    - for payoff features

- **Model Evolution**: analyze how the current market consensus (or lack of it) could change in the future and how this affects the model use.

- **Review Model Reserve/Limits**, set restrictions - monitoring (triggers)
Agenda

- What is Model Risk
- Model Risk in Past Crises
- Practical Examples
  - In Credit Modelling
  - In Interest Rate Modelling
  - In Equity Modelling
- Managing Model Risk:
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- Model Risk from 3 points of view
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  - Regulations
- Art & Science of Model Validation
## Model Validation and Model Risk Management

1. **Model Verification**
   - Mathematics
   - Implementation
   - Computational Methods
   - Payoff Description

2. **Model Validation**
   - Calibration
   - Reasonableness
   - Market Intelligence
   - Reality Check
   - Model Comparison
     - Accept/Reject
     - Improvements
     - Provisions
     - Periodic Revisions

3. **Stress Testing**
   - Stressability
   - Computational methods
     - A Payoff features
     - B Market conditions
   - Model assumptions
     - A Payoff features
     - B Market conditions
   - Improvements
   - Provisions
   - Triggered Revisions
   - Payoff Limitations
   - Model Evolution
     - Provisions
     - Triggered Revisions
Other issues in Model Risk Management

- **Approximations**
  Testing approximations against numerical methods and numerical methods against analytic results, finding special cases where comparison is possible, but that are not too special, is a crucial art in model risk management.

- **Calibration**
  Calibration can show instabilities that can be measured and tested, through ill-posedness tests and forward pricing tests. At times calibration choices are crucial to reduce model uncertainty.

- **Extrapolations**
  A delicate procedure at the heart of modelling. When they appear too arbitrary, further market data and/or further mathematics can help reducing arbitrariness.

- **Correlations**
  Difficult under a mathematical point of view, complex structure with crucial constraints like positive semidefiniteness. Very prone to structural errors (zero-correlation and one-correlation errors).
Other issues in Model Risk Management

- **Payoff**
  The payoff of index options was treated in a wrong way for years. And even now, it is still unclear, given ISDA documentation, if in case of default one receives a risk-free (Eonia discounted) residual NPV or one that includes funding costs and credit spread of survived counterparty. Dramatic consequences in contagion (Brigo and Morini (2011)).

- **Model Arbitrage**
  Can we use our no-arbitrage models to make arbitrage trades? Arbitrage trades are bets on the evolution of model uncertainty, and the management of this issue is crucial for success.

- **Hedging**
  Can be a tool for model validation, and at the same time it requires its own model risk management. Real hedging does not follow model prescriptions, in fact: traders introduce adjustments that anticipate model recalibration.

**It can also be used to quantify Model Uncertainty: see next part of Webinar**
Thank you!
The main references is the book:

* This presentation expresses the views of its authors and does not represent the opinion of his employers, who are not responsible for any use which may be made of its contents.
Part II
Model Risk Management:
Quantifying, Monitoring and Mitigating Model Risk

Model Risk in Option Pricing
Approaches to Quantifying and Monitoring Model Risk

Dr. David Eliezer, PhD, VP, Head of Model Validation, Numerix
Model Risk For (Illiquid) Derivatives

• We have seen two approaches to model risk
  – The Derman “Value” approach
  – The Rebonato “Price” approach

• The feasibility of these two approaches depends upon the liquidity of the instrument, and whether there is a long, reliable track record of two-sided market prices (liquid, vanilla instruments), or not (exotic derivatives, structured notes).

• In the case of illiquid derivatives, the Rebonato approach may not be feasible.

• Q: What can the meaning of Derman’s “value” be?
• A: It means pricing by cost of replication, with a dynamic hedging strategy.
What Do We Mean by “Model Risk” in Derivative Pricing?

• The Problem: We want to calculate a number, analogous to market risk, that measures the “model risk” of a model
  – Normally, we make a model, and use it to compute risk. How can a model compute the risk that it, itself, is the wrong model?

• Market risk can be thought of as a range of reasonable future prices. To make a similar definition of Model Risk, we seek a definition of model risk that is a range of reasonable prices, in some form. (See, for example Sibbertsen, Stahl, and Luedtke, 2009)

• To be useful to regulators and auditors, model risk needs to be expressed in either currency units or log currency units.
  – Because we would like to aggregate model risk with other risks, and other risks, e.g. market risk, are measured in price, or log price, or standard deviations thereof.
  – Because we need to compare risks to returns, and returns are measured in increments of the price, or log price.

• Whatever definition of Model Risk we settle on must give stable answers over time.
A simple approach: Survey of Market Standard Models

• If a benchmarking library of sufficient richness is available, we can estimate the range of reasonable prices by a survey of all of the market-standard pricing methods.
  
  – The survey should include a variety of models
  – The survey should include a variety of calibration methods for each model
  – Where data proxying is used, the survey should include all plausible data proxies.
  – The survey should also measure the sensitivity of each calibration method, in each model, to perturbations in the volatilities.

• This method relies on the assumption that standard market models explore the full range of reasonable, i.e. replicatable, prices. This need not be true, and true range might be wider. This method requires human judgment to set up, and is subject to criticism from regulators.
Standard Deviation of the Cost of Hedging Measure

• First suggested by Bakshi, Cao and Chen, 1997, as a way to distinguish the relative merit of different models.

• The Cost of Hedging Measure of Model Risk measures the properties of the model that practitioners care most about. It uses a procedure that practitioners have evolved over the 30 year history of the industry.

  – Traders test their models by PnL attribution, which estimates how much of the change in the option price is due to the hedge. The remainder is the “hedge error”.

  – The hedge errors may be summed up to find the cost of hedging, which is close to the upfront model price, if the model is good.

  – The hedge error is a random variable, but according to theory, it is riskless, i.e. zero variance.
Standard Deviation of the Cost of Hedging Measure

• The Cost of Hedging Measure measures the properties of the model that practitioners care most about. It uses a procedure that practitioners have evolved over the 30 year history of the industry.

  – The fair price of an option is supposed to be its cost of hedging, which is riskless, in the ideal case. In real life, the cost of hedging is not riskless, but much reduced in risk compared to, e.g. option returns.

  – If the PnL Attribution is carried out over many paths, the cost of hedging on each path may be compiled into a probability distribution of costs of hedging.

  – The maximum and minimum values of the cost of hedging provide a range of reasonable prices.

  – Prices outside of this range are arbitrage-able.

  – An arbitrage-able range of reasonable prices is enforceable by trading, and is therefore more “real” than other definitions.
Model Risk from Cost of Hedging Measure.

• We may choose the upper and lower limits for a reasonable option price, by choosing a probability threshold. This corresponds to an upper and lower allowable price, through the cost of hedging probability distribution.

• The range of reasonable prices calculated in the Cost of Hedging Measure is enforceable by an arbitrage argument -- if an option is offered for sale or purchase outside that range, then with high probability, the model tested provides a way to arbitrage that price.
Standard Deviation of Cost of Hedging Measure – Advantages/Disadvantages

• The Cost of Hedging Measure may be computed without human labor or human judgment, but with computation time instead. It is automatable.

• The path construction algorithm requires making some form of stationarity assumption. The test must be run with a variety of these assumptions, to show that the conclusions do not depend on the assumptions.

• This measure of model risk captures many of the most difficult sources of model risk.
  – Recall the issues on which the Derman Model Risk is focused – misspecification of the stochastic dynamics
  – If a model has been mis-specified, or mis-calibrated, or has been used with poor market data inputs, all these issues will lead to poor hedging, and so will show up in the computation as a more variable cost of hedging. i.e. larger model risk.
Monitoring Model Performance with Daily Hedge Error Distribution

• The daily hedge error distribution can be computed from the hedge errors on each day of each path.

• This distribution can be used to monitor the current performance of the model, by evaluating the probability of realizing today’s hedge error, given that the model is just as correct as it was when the model was tested.

• The probabilities across several days can be strung together into a joint distribution for hedge errors.
  – We may compute the probability of any realization of hedge errors that we see in production, assuming the model is correct.
  – When this probability falls so low that the correctness of the model is implausible, we may trigger a re-examination of the model.
Conclusions

- Model Risk must be defined as the range of reasonable prices, in order to be aggregated with other sources of risk.

- Standard methods exist, but require human labor to perform them, and are hard to defend theoretically. This may lead to drawn out debates with regulators/auditors, and a long, slow validation process.

- Alternatively, we may compute width of the distribution of the cost of hedging as a measure of model risk. This is grounded in arbitrage pricing theory, and so is easier to defend, and allows much less room for debate. And it may be computed straightforwardly, without human intervention.

- The daily hedge errors distribution can be used to monitor the validity of the model on a day-by-day basis. It also is grounded in arbitrage pricing theory, and is easily defended to regulators.
Submit Your Questions Now....

Click the **Q&A Button** on the **WebEx Toolbar** located at the top of your screen to reveal the **Q&A Window** where you can type your question and submit it to our panelists.

Q: Your Question Here

A: Typed Answers Will Follow or We Will Cover Your Question During the Q&A At the End

Note: Other attendees will not be able to see your questions and you will not be identified during the Q&A.

We will provide the slides following the webinar to all attendees.
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